Solar Sail Enabled Near-Vertical Earth-Trailing Orbits

James B. Pezent, Rohan Sood, Andrew F. Heaton

Abstract
Recent advancements in solar sailing technology and the upcoming 2020 launch of NASA's Near-Earth Asteroid Scout mission have renewed interest in expanding mission scenarios for scientific exploration. Regions that are outside the reach of traditional propulsion systems or require significant propellant may be accessed by harnessing the solar radiation pressure and leveraging coupled dynamics to maneuver the sail-based spacecraft. Earth-trailing orbits have recently been investigated for getting a unique perspective of the Sun while maintaining the spacecraft in close proximity to Earth. Near-vertical orbits trailing the Earth exhibit additional capability to view the Sun from above and below the ecliptic plane while satisfying the given constraints. Families of sail-based orbits are explored for varying Earth-trailing angles and inclinations. Optimization is carried out to ensure that the non-traditional orbits exhibit a constant pitch angle control history that maximize solar observation.

Keywords: Solar Sail, Vertical Orbits, Artificial Equilibrium Points, Earth-Trailing Orbits, Solar Observation

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\mathbf{r}} )</td>
<td>Sail Normal Vector</td>
</tr>
<tr>
<td>( \mathbf{\ddot{a}}_s )</td>
<td>Sail Acceleration Vector</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Sun Incidence Angle</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Sail Clock Angle</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Sun-Earth Mass Parameter</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Sail Lightness Parameter</td>
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1. Introduction

As a successor to the Near-Earth Asteroid (NEA) Scout spacecraft, a sail-based mission to a near-Earth asteroid, there is a growing interest in leveraging solar sails for heliophysics science missions. Past studies supporting the now-cancelled Sunjammer solar sail mission extensively examined both sub-L\(_1\) and Earth-trailing equilibrium points as potential destinations conducive to space-weather forecasting and solar observation [1,2]. In response to the possibility of a new mission, previous work examined planar orbits about close Earth-trailing equilibrium points. Trajectories were specifically designed to exhibit zero incidence angle variation in an effort to both maximize solar observation and simplify sail attitude control [3]. In this work, Earth-trailing trajectories are developed that exhibit near-vertical motion while retaining a constant pitch angle control history. Such non-traditional orbits allow for solar observation from out of the ecliptic plane, and can be sustained at Earth-trailing angles where equilibrium solutions are dynamically infeasible.

2. Dynamical Modelling

Preliminary trajectories are designed within the framework of the Sun-Earth circular restricted three-body problem (CR3BP). The spacecraft is modeled as a massless object under the gravitational influence of the Earth and the Sun, which are both assumed to be moving in circular orbits about their barycenter. The equations of motion are expressed in the Sun-Earth rotating frame, and units are non-dimensionalized such that the non-dimensional distance between the Earth and Sun is unity, and the period of the system is 2\( \pi \). The resulting scalar expressions, shown in Eq. 1, govern the acceleration of the \( x \), \( y \), and \( z \) components of the spacecraft’s state relative to the barycenter of the system.

The additional quantities, \( r \) and \( d \), represent the scalar distance of the spacecraft from the Sun and Earth respectively, while the mass parameter, \( \mu \), is the ratio of the Earth’s mass to that of the total system [4].

Though they appear simple, the dynamics are chaotic and exhibit no closed form solutions, but five equilibrium points, or Lagrange points, are known. The periodic/quasi periodic orbits that exist in the vicinity of the Lagrange points can be leveraged to place a

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spacecraft in an orbit about a fixed location or to maneuver the craft along the associated manifold for cost-effective transfers [4-6].

\[
\begin{align*}
    a_{gx} &= 2y + x - (1 - \mu) \frac{x + \mu}{a^3} - \mu \frac{x + \mu - 1}{r^3} \\
    a_{gy} &= -2x + y - (1 - \mu) \frac{y}{a^3} - \mu \frac{y}{r^3} \\
    a_{gz} &= -(1 - \mu) \frac{z}{a^3} - \mu \frac{z}{r^3}
\end{align*}
\]

(1)

A small, but constant thrust provided by a solar sail is well suited to exploit the complex dynamics arising within the CR3BP, as shown by numerous researchers [7-12]. For an ideal solar sail, the effects of the solar radiation pressure (SRP) are modeled by incorporating an additional acceleration term along the sail-normal direction, \( \hat{n} \), to Eq. 1, and is expressed in a vector form by Eq. 2.

\[\vec{a}_s = \vec{\beta} \left( 1 - \frac{\mu}{|\vec{a}|^2} \cos^2(\alpha) \right) \hat{n}\]

(2)

The solar sail orientation angle, \( \alpha \), measures the angle of incidence/pitch between the solar radiation pressure vector and the sail’s normal vector, \( \hat{n} \) as illustrated in Fig. 1. The clock angle, \( \gamma \), can be implicitly defined using Eq. 2 as the rotation of the pitched normal vector about \( \vec{d} \) relative to the \( \vec{Z} \) axis. The constant \( \beta \) term, also known as the lightness parameter, is the main performance metric of the sail and is a function of the mass and the sail area. In general, higher \( \beta \) will confer increased maneuverability and lower transfer time of flight (TOF).

With technological advancement, next generation solar sails are projected to have a \( \beta \) value ranging from 0.02 to 0.03 [13]. As such, values within this range are used as a baseline for orbit analysis.

3. Earth-Trailing Equilibrium Points

In the Sun-Earth rotating frame, the simplest class of close Earth-trailing trajectories are the solar sail equilibrium points first explored by McInnes and developed further by Heiligers and Farres [1,2,11,12]. If properly positioned, a solar sail can remain stationary in the Sun-Earth rotating frame using a constant \( \alpha \) angle. Placing a solar sail at a trailing equilibrium point is an attractive option from the standpoint of pointing onboard scientific instruments. By pre-selecting a desired trailing angle, \( \theta_t \), for a mission, solar imaging equipment can be bore-sighted off the sail’s normal vector by an equilibrium \( \alpha \) angle, thus allowing the spacecraft to orient itself for constant solar observation. Likewise, communications antennae may be simultaneously aligned to stay in constant contact with Earth, reducing the need for dedicated communication slews. Following the analysis of McInnes, the \( \beta \) value and \( \hat{n} \) necessary to maintain an arbitrary equilibrium point within the CR3BP are analytic functions of the sail’s position within the Sun-Earth rotating frame and can be computed using Eq. 3 and Eq. 4, respectively [11].

\[\beta = \frac{1}{(1 - \mu) \cos^2(\alpha)} \left( \frac{|\vec{\beta}|}{|\vec{\beta}_g|} \right)\]

(3)

\[\hat{n} = -\frac{\vec{\beta}_g}{|\vec{\beta}_g|}\]

(4)

However, it is necessary to exclude locations such that \( \cos(\alpha) \) is less than zero since the sail cannot provide thrust in the direction of the Sun. Using Eq. 3 and Eq. 4, the planar Earth-trailing configuration space can be sampled to produce contours of constant \( \beta \) equilibrium points. For any given \( \beta \) value, the solutions where \( \theta_t \) is equal to 0° and 60° can be maintained with \( \alpha = 0^\circ \), and correspond to the extensively studied collinear sub-L1 and triangular sub-L5 points, respectively. As noted by McInnes, when \( \alpha \) is non-zero, equilibria are not Lyapunov stable. Hence, the eigenvalues of the linearized equations of motion lose some of their predictive power when \( \theta_t \) is not equal to 0° or 60°. However, practical information regarding the stability and period of nearby motion may still be obtained. An approximate stability index, \( \lambda_r \), can be defined as the average of the absolute value of the real parts of each eigenvalue [3]. Larger values of \( \lambda_r \) indicate a greater tendency to diverge from the equilibrium point. Furthermore, the approximate period of nearby trajectories can still be inferred from the eigenvalues associated with the most stable subspace. The stability index (color bar) and equilibrium contours (white lines) for \( \beta \) values ranging from 0.01 to 0.05 are plotted in Fig. 2 for the dynamically rich Earth-trailing angles of 0° to

![Fig.1. Visual illustration of solar sail orientation angles and vectors shown in Eq. 3.](image-url)
It is evident from Fig. 2 that a bifurcation in the overall behavior occurs at a critical lightness number, between 0.025 and 0.03 ($\beta = 0.0281$). For $\beta$ above 0.0281, equilibrium contours can be extended smoothly from sub-$L_1$ to sub-$L_5$ and exhibit a relatively high stability index in the bounded region between the two red lines. When $\beta$ is below the critical value ($\beta < 0.0281$), contours extending from the corresponding sub-$L_1$ point loop back towards the vicinity of the traditional $L_1$ point. A second contour then extends from the sub-$L_5$ point to a $\beta$-dependent minimum trailing angle near the Earth before turning back towards $L_5$. As a result, there is a dead-zone where it is dynamically impossible to sustain an artificial equilibrium point which excludes the sail from being placed in the high-stability, low-trailing angle region. However, regardless of the $\beta$ value, equilibrium solutions exhibit sharp increase in stability for trailing angles beyond roughly 2.5° (light-blue line), indicating the edge of the Earth's sphere of influence (SOI) with respect to statically aligned sails.

4. Planar Earth-Trailing Orbits

Extensive investigations in the traditional CR3BP and in practice have shown that greater long-term stability can be achieved by orbiting rather than remaining stationary at an equilibrium point. Within the context of a stereoscopic solar imaging mission constrained by Earth communications distance, orbiting also delivers additional benefits. An extended orbit allows the spacecraft to observe the Sun from a wider range of trailing angles with respect to Earth-based observatories. Additionally, depending on the trailing angle and orbit size, communication range with Earth is reduced for one half of the orbit’s period.

In a previous investigation, the motion about Earth-trailing solar sail equilibrium points was extended into a new class of planar, controlled periodic orbits maintained by a near-optimal station-keeping strategy [3]. Starting with a given trailing angle, $\theta_t$, and $\beta$ value, the period of a nearby orbit is inferred from the frequency of the short period mode of the associated equilibrium point. An initial guess for the controlled periodic orbit is constructed from the state history consisting of multiple copies of the equilibrium point state and control, with all arc times summing to the estimated period.

The initial guess is then passed to a direct transcription algorithm constrained to enforce periodic motion and control [3,7,8]. An additional constraint is also applied to produce a fixed orbital amplitude, $A$. This is enforced by constraining the initial state to lie at a distance, $A$, along the vector extending from the equilibrium point towards the system's barycenter. Depending on the trailing angle, this roughly corresponds to constraining the semi-minor axis of the orbit as viewed in the Sun-Earth rotating frame, or the eccentricity if viewed in an inertial frame. To ensure the orbits exhibit minimal $\alpha$ variation, it is necessary to optimize periodic trajectories with a carefully chosen objective function. To achieve this, the sum of squared deviations between the cosine of the $\alpha$ angle at each state in the trajectory and a single free pitch angle, $\alpha_f$, is minimized. As shown in Eq. 5, minimizing a discrete sum of the squared deviations is selected instead of the integral of squared deviations to prevent the orbit from collapsing the trajectory to a single point.

$$\sum_i^n (\hat{d}_i \cdot \hat{n}_i - \cos(\alpha_f))$$

When solved with a small non-zero value of $A$, the solution expands from the equilibrium point into a closed orbit. A continuation scheme is applied to produce a family of orbits by increasing the amplitude of the semi-minor axis and seeding with a previously converged orbit. An example of a subset of a fully...
A converged family at a trailing angle of 3° and β value of 0.025 is shown in Fig. 3.

Fig.3. Planar 3° Earth-trailing family of orbits.

The optimization continuation process is successful in producing a family of nearly-planar orbits that exhibit zero α variation. Station keeping is instead accomplished entirely through osculation’s in the sail’s clock angle near periapsis. Previous results have shown that it is possible to construct similar orbit families for β values ranging from 0.01 to 0.05 at trailing angles of 1.5° to 15° [3].

5. Near-Vertical Earth-Trailing Orbits

Analysis of planar solutions has since been extended to explore near-vertical orbits in the close Earth-trailing regime. Similar to the planar case, an initial guess for the state and control is taken to be the equilibrium point at a desired trailing angle, however, the initial guess for the period is now inferred from the frequency of the apparent vertical motion. Additionally, the constraint on the planar amplitude, A, is removed and substituted with a constraint that fixes a positive vertical amplitude, Z, of the initial state. The optimization continuation process then proceeds to produce a family of orbits parameterized by increasing the Z amplitude. The formulation utilizes the same objective function defined by Eq. 4 that minimize variations in α.

The resulting orbit families depend on the β value as well as the initial equilibrium point. In general, all computed sail-based orbits, thus far, exhibit similar characteristics and can be sustained with a single pitch angle. Further insight into the overall behavior can be gained by examining a rather typical case with a β value of 0.02 extending from a 2° Earth-trailing equilibrium point (Fig. 4,5). The orthographic and the Y-Z projection of the sail-based family of orbits is shown in Fig. 4. Note, all figures are colored coded based on the Z amplitude of the corresponding orbit, and the axes have been visually scaled for clarity.

Fig.4. Earth-trailing vertical orbit family (top: orthographic view; bottom: Y-Z projection). [Axes scaled for clarity]

The blue diamond (2° EP) indicates the artificial equilibrium point trailing the Earth by 2° relative to the Sun-Earth line. The Y-Z projection (bottom Fig. 4) shows that the sail-based periodic orbit family is not symmetric about the X-Y plane. The X-Y projection in
Fig. 5 (top) shows how the family grows relative to the 2° trailing artificial equilibrium point. Whereas, the bottom plot in Fig. 5 illustrates the out-of-plane motion of the orbits. Once again, note that the orbits are not symmetric about the X-Y plane.

To further assess the control history, it is vital to investigate the evolution of the two control angles, $\alpha$ and $\gamma$. Fig. 6 shows the corresponding constant pitch angle, $\alpha_r$, as a function of the orbital amplitude (top) as well as the clock angle, $\gamma$, control history as a function of the orbital period (below). Orbits with small Z amplitudes, based on the color bar, tend to remain near the equilibrium point sustained by small sinusoidal variations in the clock angle. As the Z amplitude increases (0.02-0.05 ND), optimal solutions exhibit a consistent figure-8 shape and begin to drift towards the bounded high stability region in Fig. 2 (between the two curved red lines). The equilibrium pitch angle stays relatively constant at 6.6°, however, the amplitude of sinusoidal $\gamma$ variations become relatively more pronounced as evident from bottom plot in Fig. 6. When the solutions begin to cross over the bounded stability region, a near symmetric sinusoidal $\gamma$ control history is no longer sufficient to maintain periodicity, and sharp changes are seen in the equilibrium pitch angles. Note, for the illustrated $\beta$ value of 0.02, planar orbits about the artificial equilibrium points at these trailing angles are dynamically infeasible. However, the addition of the out-of-plane motion, coupled with solar sail dynamics, lowers the perturbing effects of the Earth, enough to allow for orbits that osculate vertically in this region. As the Z amplitude further increases, adjacent sail-based solutions stop drifting towards the Earth and begin to climb vertically at a trailing angle of 0.5°. Subsequently, the equilibrium pitch angle remains relatively constant and solutions are visually similar to traditional vertical orbits about L1 that have been shifted to trail the Earth. The family can be extended to higher Z amplitudes than what is shown (0.1 ND), but solutions retain the overall shape and control history, similar to the highest amplitude orbit shown in Fig. 4-6.
6. Conclusion

Results, thus far, indicate that near-future solar sails can enable near-vertical orbits in the close Earth-trailing regime. Similar to the classical solar-sail equilibrium points and previously explored planar solutions, sail-based periodic orbits can be maintained using a single sail pitch angle. However, vertical orbits can allow for out of the ecliptic solar observation and can also be placed at Earth-trailing angles inaccessible to solar sails with smaller $\beta$ values. Future work will focus on refining the continuation process used to generate and parameterize vertical orbit families. Specifically, additional constraints on orbital symmetry, a simplified control parameterization, and high fidelity sail models will be investigated.

References


